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Optimization of congested traffic by controlling stop-and-go waves

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We propose a new optimization strategy based on inducing stop-and-go waves on the main road and controlling their wavelength. Using numerical simulations of a recent stochastic car-following model we show that this strategy yields optimization of traffic flow when implemented in systems with a localized periodic inhomogeneity, such as signalized intersections and entry ramps. The optimization process is explained by our finding of a generalized fundamental diagram (GFD) for traffic, namely a flux-density-wavelength relation. Projecting the GFD on the density-flux plane yields a two-dimensional region of stable states, qualitatively similar to that found empirically [Kerner, Phys. Rev. Lett. **81**, 3797 (1998)] in synchronized traffic.

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Traffic flow has been a subject of comprehensive study for more than half a century [1–20] due to its theoretical and practical importance. Recently this field has attracted much interest, especially after new empirical and theoretical studies have shown its clear relation to physical phenomena of current interest, such as phase transitions, critical phenomena, nonlinear dynamics, and chaos (for reviews see, e.g., [1–3]).

One of the main open questions in this field regards the validity of the basic concept of the "fundamental diagram" [4–6]—a functional relation between the flux and the density of cars. This generally recognized relation, used in almost every study in this field, was challenged recently by empirical findings of Kerner [7,8] indicating that such a fundamental diagram does not exist. Instead, stable synchronized traffic states display a two-dimensional region in the density-flux plane. Consequently, the wide scattering of data points representing congested traffic cannot be attributed only to measurement fluctuations, but also to the existence of a range of stable flux values for a given density. The existence of a range of flux values gives rise to the possibility of manipulating the system in order to achieve the highest possible flux.

In order to examine this possibility of optimization, we choose to study systems with a localized periodic inhomogeneity. Two types of periodic inhomogeneities are considered: (a) signalized intersections and (b) entry ramps with signalized entrances. We focus on cases of oversaturation, i.e., where traffic is congested upstream of the inhomogeneities. These systems are studied using numerical simulations of a recent car-following model [9]. In this model, as well as in other microscopic models [10–16], traffic is treated as a flow of interacting particles. In inertial car-following models [9,10,13–15] this interaction is defined by a relation between the acceleration a of a car, its headway Δx, velocity v, and velocity difference with the car ahead Δv, i.e., $a = a(\Delta x, v, \Delta v)$. In particular, for the model we use [9]

$$a = A\left(1 - \frac{\Delta x_0}{\Delta x}\right) - \frac{Z^2(-\Delta v)}{2(\Delta x - D)} - kZ(v - v_{per}) + \eta', \quad (1)$$

where D is the minimal, and $\Delta x_0 = vT + D$ the optimal, distance to the car ahead. The constant T is a safety time gap, A, k, and v_{per} are constants, and the function Z is defined as $Z(x) = (x + |x|)/2$. More details can be found in [9]. In the numerical solutions of Eq. (1), the random term η' is realized by choosing for each car a random number uniformly distributed in the range $-0.5\eta \leqslant \eta' \leqslant 0.5\eta$ every time step. Here we use the same choice of parameters as in [9] with $a = 3$ m/sec^2 and a numerical time interval $\Delta t = 0.1$ sec.

(a) *Signalized intersection*. Signal optimization theories [4,21–24] are usually concerned with optimizing global quantities such as the total delay time of all drivers in the system. Our aim here is different—to optimize the flux in a given direction of an intersection. For simplicity, we consider a single direction (as, e.g., in [21]) and we assume that the intersection is occupied by vehicles coming from other directions for a given portion P_r of the total signal period τ. The rest of the signal period is divided into three parts: τ_g and τ_y are the durations of the green and yellow [25] lights, and τ_- is a given additional safety red light period. Thus $\tau = P_r\tau + \tau_g + \tau_y + \tau_-$. The average flux $f(\tau)$ out of the considered direction is expected to be [21]

$$f = f_0\left(1 - P_r - \frac{\tau_-}{\tau}\right), \quad (2)$$

where f_0 is the constant flux during the green light period. Since usually $\tau \gg \tau_-$, the flux should be hardly influenced by τ.

However, the results of our simulations shown in Fig. 1 are different from Eq. (2) for both stochastic and deterministic models. Apart from trivial flux oscillations which are related to the fact that all cars have identical parameters [26], Fig. 1 shows a monotonic increase of the flux f as τ grows for the deterministic model $\eta = 0$. For the stochastic model $\eta > 0$, however, an optimal signal period can be clearly seen. In this case, a crossover is observed from the deterministic monotonically increasing $f(\tau)$ values to a saturated lower value of the flux of large τ. Therefore there exists an optimal signal period that yields maximal flux. Such influence of τ on f is not explained by Eq. (2).

In an attempt to explain these unexpected results, we perform extensive simulations of the deterministic model (1) on

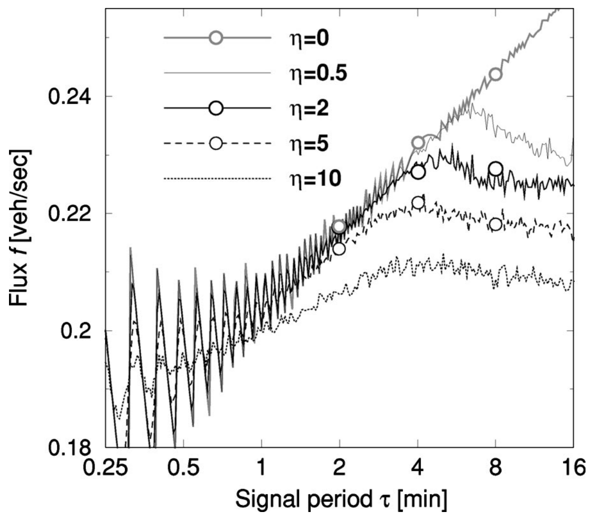

FIG. 1. Relation between the signal period and flux for the values of acceleration noise amplitude $\eta=0,0.5,2,5,10$ m/sec^2 (top to bottom). Traffic light parameters are $P_r=1/3$ and $\tau_y=\tau_-=2$ sec. The total number of cars in the system is $N=400$ and its length is $L=10$ km. The nine open circles correspond to the nine instances presented in Fig. 3. Both the increase of $f(\tau)$ for $\tau \gg \tau_-$ in the deterministic case ($\eta=0$), and the existence of an optimal τ for the stochastic model ($\eta>0$) are unexpected according to (2).

homogeneous systems with periodic boundary conditions, starting from different initial conditions [27]. Figure 2(a) presents the flux measured for the steady states of this model (see [9]): stop-and-go waves (surface) and homogeneous free and congested flow (thick lines). The projection of the surface in Fig. 2(a) on the density-flux plane [thin curves in Fig. 2(a)] provides a two-dimensional region of stable states, qualitatively similar to that found empirically [7,8] for synchronized flow. Thus, in contrast to the common belief that the flux depends only on the density of cars (fundamental diagram), we obtain a generalized fundamental diagram [Fig. 2(a)] which shows that the flux depends on two variables—density ρ and wavelength λ.

A typical relation between the flux and the wavelength of stop-and-go states for a fixed density is presented in Fig. 2(b). The simulations, furthermore, show that these states become unstable as the noise η is increased above a certain threshold. The dependence of the instability threshold η_{th} on the wavelength λ is shown in Fig. 2(c).

The existence of an optimal signal period can now be explained using our finding of the influence of λ on the flux and on η_{th}. First, it is easy to see that the signal period τ controls the wavelength λ of stable stop-and-go waves that are induced by the traffic lights, as $\lambda=v\tau$, where v is the wave velocity [6]. From Fig. 2(b) it appears that in order to increase the flux, λ—and therefore τ—should be increased [28]. For this reason the deterministic model yields a monotonically increasing $f(\tau)$ (Fig. 1, upper curve). However, for the stochastic model ($\eta>0$), states for which $\eta_{th}(\lambda)<\eta$ become unstable according to Fig. 2(c)—in particular those with relatively high values of λ, which in principle should yield higher values of flux. The increase of the flux with increasing τ thus crosses over to lower values due to the

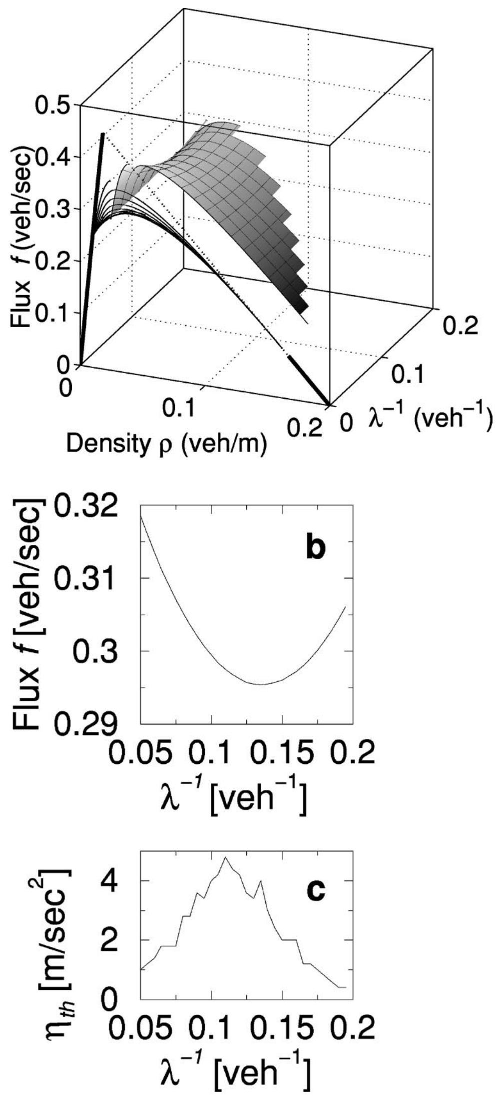

FIG. 2. (a) The generalized fundamental diagram (GFD): density-wavelength-flux relation for the different states of the deterministic model on the homogeneous system with periodic boundary conditions. Shown are results obtained for different states of the deterministic model: stable stop-and-go waves (surface), and stable and unstable homogeneous states (thick and dotted lines, respectively). The wavelength is defined as the average number of vehicles between centers of nearest dense regions in stop-and-go traffic flow. A number of curves with fixed wavelength ($\lambda^{-1}=2/60,3/60,\ldots,12/60$) are projected on the density-flux plane (thin curves). (b) A cross section of (a) for a density $\rho=0.06$ vehicles/m, demonstrating the typical dependence of the flux on the wavelength. (c) Noise stability threshold amplitude η_{th}, above which the states presented in (b) become unstable. From the two latter figures it follows that in order to optimize stop-and-go traffic in the presence of noise, it is necessary to increase the wavelength up to some optimal value, above which the stop-and-go waves would become unstable.

instability of the induced waves. The existence of an optimal τ is a result of this effect.

To visualize this process, nine space-time diagrams are plotted in Fig. 3(a), in correspondence to the nine instances

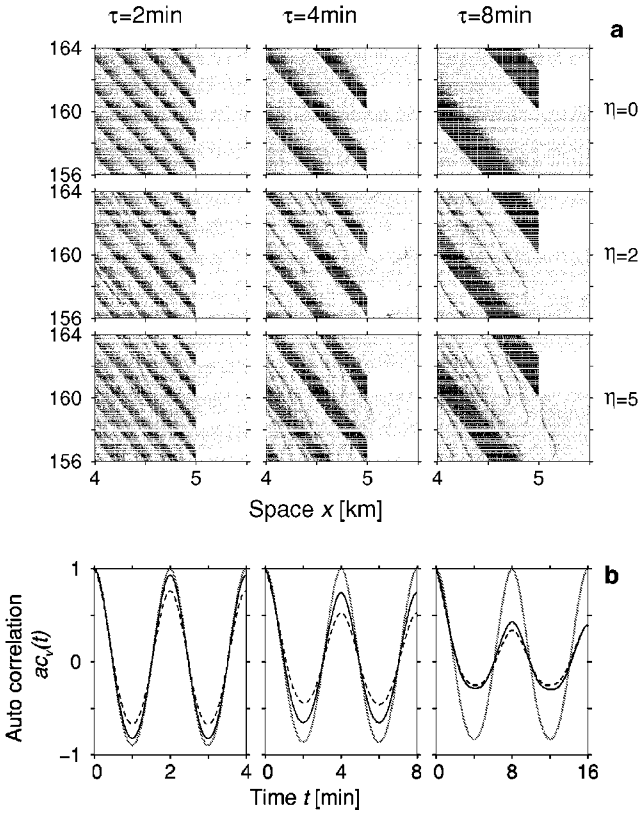

FIG. 3. (a) Space-time diagrams and (b) autocorrelation functions of systems with single traffic light with parameters as the nine instances denoted with open circles in Fig. 1. The position of the traffic light is at $x=5$ km. The major dense (black) regions moving upstream in (a) are caused by the red light, and the distance between such nearest two regions (corresponding to the wavelength λ) is growing as τ grows. The gray curves in (b) correspond to $\eta=0$, solid curves to $\eta=2\,\mathrm{m/sec}^2$, and dashed curves to $\eta=5\,\mathrm{m/sec}^2$. A comparison to Fig. 1 shows that the crossover to reduced values of flux is related to the emergence of small jams, which causes loss of the periodicity and reduced values of the autocorrelation function, $ac_v(t=\tau)<1$.

denoted by circles in Fig. 1. Each dot in the space-time diagrams represents the position of a single car at a certain time, so that the dark regions represent dense regions on the road. The increase of the dominant wavelength with increasing τ can be easily seen in this figure. But unlike the deterministic case ($\eta=0$) where the flow is highly periodic, in the stochastic model ($\eta>0$) small jams emerge in the regions of low density, when the noise amplitude η or the signal period τ exceed certain thresholds. In this case other values of wavelength which are smaller than that induced by the traffic light, are effectively involved. The crossover in $f(\tau)$ to lower values observed in Fig. 1 for the stochastic model is related to the appearance of these small jams.

To obtain further support of this explanation we evaluate a measure for the periodicity of the flow using single vehicle data collected at the intersection. We calculate the autocorrelation function $ac_v(t)$ [17] of the velocity function $v(t')$ measured at the intersection,

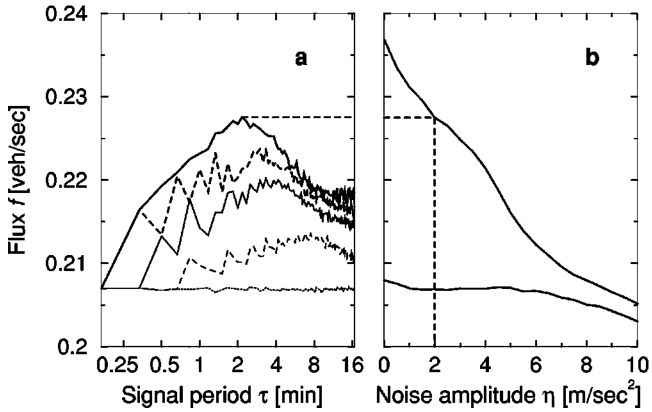

FIG. 4. (a) Relation between a signal period on the entry ramp and flux on the main road, for $P_r=0.7,0.5,0.4,0.2,0.0$ (top to bottom), and for noise amplitude $\eta=2\,\mathrm{m/sec}^2$. The total number of cars in the system is $N=300$, the system length is $L=10$ km, and the flux is locally measured on the main road at 100 m upstream to the on-ramp. Here $f_{in}=0.1$ vehicles/sec and $f_{max}=0.333$ vehicles/sec. (b) A comparison between the optimal flux (upper curve) and the flux without the presence of a traffic light (lower curve), as a function of η.

$$ac_v(t)=\frac{\langle v(t')v(t'+t)\rangle-\langle v(t')\rangle\langle v(t'+t)\rangle}{\langle v(t')^2\rangle-\langle v(t')\rangle^2}, \qquad (3)$$

where a linear interpolation of the discrete function $v(t)$ is used. The brackets $\langle\cdots\rangle$ indicate averaging over time t'. Displayed in Fig. 3(b) are the autocorrelation functions for the nine instances of Fig. 3(a), respectively. As can be seen from this figure, $ac_v(t=\tau)=1$ for all the $\eta=0$ instances, implying that the flow for these cases is completely periodic, and that the time period of $v(t)$ is exactly τ. A comparison of all instances shown in Fig. 3(b) to the data in Fig. 1 shows that the flux approaches the deterministic value as long as $ac_v(t=\tau)\approx 1$. As η or τ are increased, the flow is no longer periodic [$ac_v(t=\tau)<1$ in Fig. 3(b)], small jams emerge [Fig. 3(a)], and the flux becomes lower (Fig. 1).

(b) *Entry ramp with signalized entrance*. Next, we study another example of localized inhomogeneity, caused by an entry ramp [29,30]. To make this inhomogeneity periodic, we introduce traffic signals at the downstream end of the entry ramp, and study its effect on the flux on the main road. The incoming vehicles are allowed to enter the main road during the green light period τ_g, and are delayed during the red light period $P_r\tau$, so here $\tau=P_r\tau+\tau_g$. Unlike the signalized intersection where P_r was predetermined, here it is one of the optimization parameters, in addition to τ. Similar to [18], we introduce an exit ramp at a large distance from the entry ramp, so that the total number of cars in the system is conserved. The entrance and the exit of cars from the ramps are performed in the same manner as in [9]. We focus on cases where the average flux of the incoming vehicles f_{in} causes congestion on the main road (see [19,20]), but the secondary road is not congested. Since our goal is to optimize the flux on the main road without causing congestion on the secondary road, we set an upper bound for P_r. This bound is P_r

$\leqslant 1 - f_{in}/f_{max}$, where f_{max} is the maximal possible value of the incoming flux, since cars approach the queue upstream to the traffic light with rate f_{in}, and this queue is discharged with rate f_{max} during the green light.

Typical relations between flux and the signal period are plotted in Fig. 4(a) for different values of P_r and for $\eta = 2$ m/sec^2. Note that the curve corresponding to an unsignalized entry ramp ($P_r = 0$) is the lowest, implying that the introduction of a traffic light increases the flux on the main road. Moreover, the increase in the flux on the main road is obtained without causing congestion on the secondary road. The relative increase in the flux due to the introduction of a traffic light with optimal parameters [Fig. 4(b)] varies from 1.0% for $\eta = 10$ m/sec^2, through 10.0% for $\eta = 2$ m/sec^2, and up to 13.9% for $\eta = 0$. This increase of the flux f has even a more significant influence on the growth rate of the congested section of highway upstream of the entry ramp, since this rate is proportional [6] to $f' - f$, where f' is the flux upstream to this region. These results suggest that even when congestion of real traffic cannot be relieved, the flux can be increased by an optimal signalization of entry ramps to control and stabilize stop-and-go waves.

In addition to the two studied systems, the new optimization approach may also be efficient in traffic control systems that are based on varying speed limits along the road. In these systems, stop-and-go traffic waves can be controlled [31], and therefore they can also be optimized using this approach.

To summarize, this work was motivated by the experimental findings of a two-dimensional representation of synchronized flow in the density-flux plane [7,8]. Using a deterministic car following model [9] we are able to show that the fundamental diagram has to be generalized to include another variable, the wavelength of the stop-and-go waves. The projection of the generalized fundamental diagram (GFD) on the density-flux plane yields a two-dimensional region of stable states, qualitatively similar to that found experimentally for synchronized flow. We use the GFD to propose a novel strategy for traffic optimization, based on inducing stable stop-and-go waves yielding the maximal flux.

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[25] The period of yellow light is realized as follows. When the light is changed from green to yellow, all simulated drivers upstream to the intersection estimate the intersection crossing time t_n by a linear extrapolation of their position. The first car that begins to stop, s, is the first car that is not able to cross before the light changes to red, i.e., $t_s = \min\{t_n | t_n > \tau_y\}$. For this car, Δx in Eq. (1) is replaced with the distance between s and the traffic lights. The consecutive cars follow s and stop according to Eq. (1). Due to this procedure cars can still cross the intersection during time τ_y.
[26] These oscillations are caused by the discrete nature of the flowing media and by the fact that all drivers are identical. Thus, the magnitude of these oscillations is highest for $\eta = 0$, for which the crossing times in each cycle are the same, and is in the order of $1/\tau$.
[27] Interactive simulations of the deterministic model can be found at http://ory.ph.biu.ac.il/2000/traffic/
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